EPR-steering Inequalities from Entropic Uncertainty Relations

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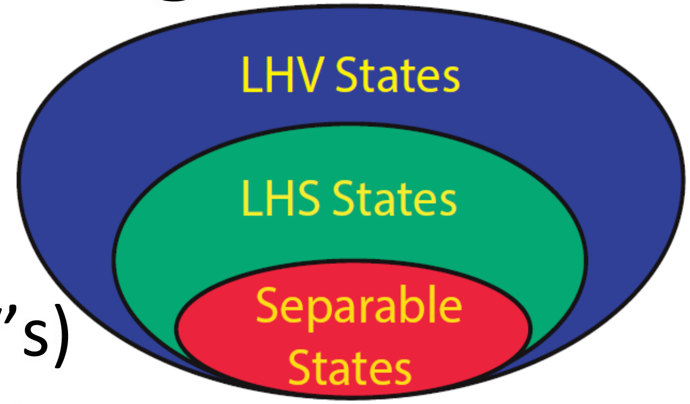
- What we have shown
  - If you have an entropic uncertainty relation…

  …Then you have a steering inequality.

- Why this is important
  - They are intuitive entanglement witnesses.
  - They are (much) easier to use than doing state tomography.

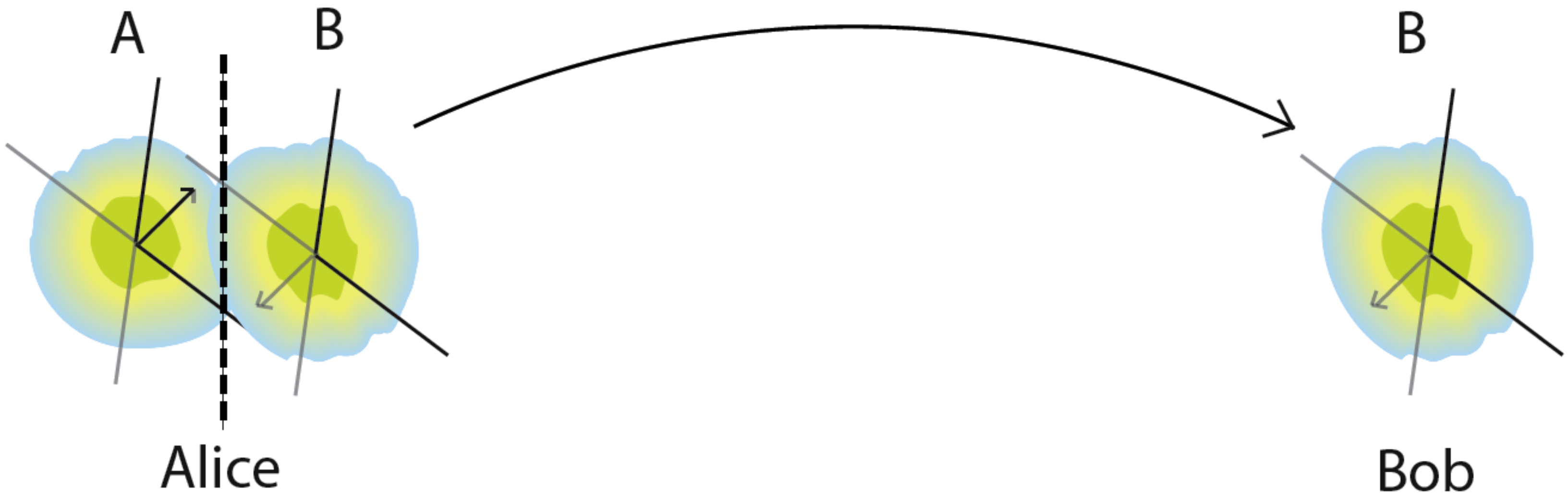
What is EPR-steering?



- It is a degree of nonlocality.
  - Bell nonlocality (all LHVs)
  - EPR steering (All LHS's, some LHV's)
  - Implies correlations strong enough to demonstrate EPR "paradox".

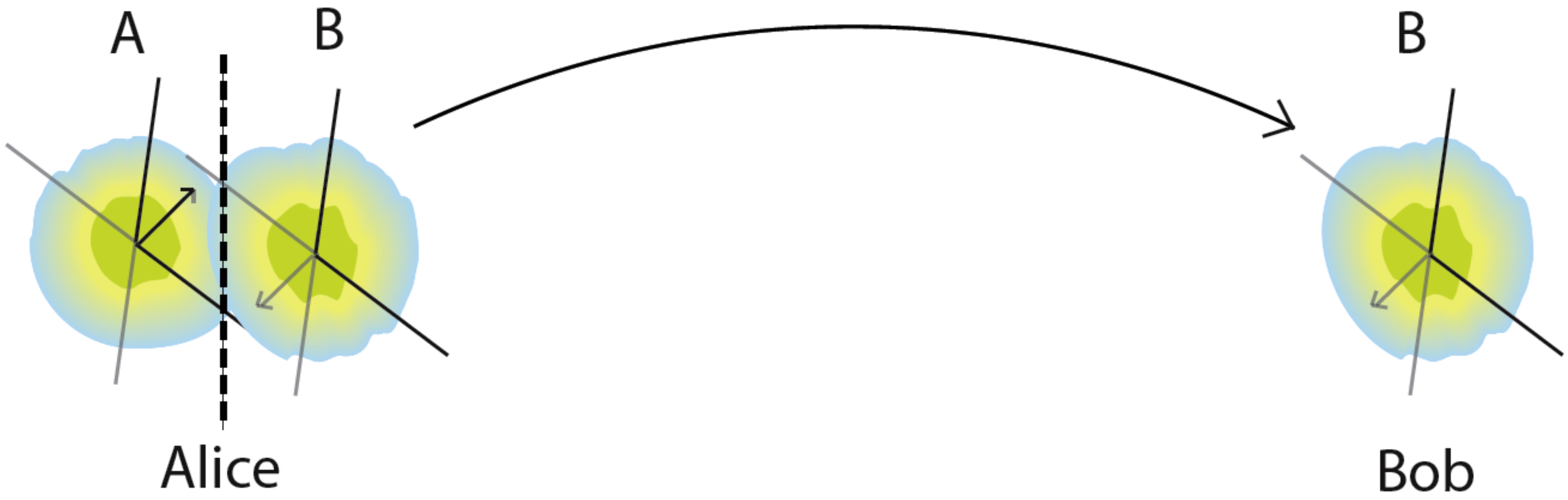
- It signifies what you can do with these correlations.
  - You can verify entanglement **even when one party's measurements are untrusted!**

The situation in EPR-steering



- Alice prepares A and B, and sends B to Bob.
- Bob tells Alice to measure ($\vec{x}$ or $\vec{p}$) of A, choosing randomly.
- Alice reports to Bob her measurements.
- Bob examines the correlations between his and her measurement results.

The situation in EPR-steering



How can Alice prove there's entanglement?

- **If Alice were preparing and sending states to Bob, the measurement correlations could only be so high**.
    - Bob could tell Alice to measure $\vec{x}$ even though she sent a state with definite $\vec{p}$.
- A **steering inequality** gives an upper limit for these local correlations.

Where do steering inequalities come from?

- Models of local hidden states (LHS):
  - Models where (Alice) is preparing and sending states to (Bob).
  - Models where (Bob's) state is known and classically correlated to (Alice's) results.
  - All LHS models for Bob have joint measurement probabilities of the form…

$$\rho(x^A, x^B) = \int d\lambda\, \rho(\lambda)\rho(x^A|\lambda)\rho_q(x^B|\lambda)$$

$$\mathrm{P}(R^A, R^B) = \sum_{\lambda} P(\lambda)P(R^A|\lambda)P_q(R^B|\lambda)$$

From Uncertainty to EPR-steering

From the relative entropy between $\rho(x^B,\lambda|x^A)$ and $\rho(\lambda|x^A)\rho(x^B|x^A)$being ≥ 0, we get

LHS constraints:

- Continuous variable [2]:

$$h(x^B|x^A) \geq \int d\lambda \rho(\lambda) h_q(x^B|\lambda)$$

- Discrete variable [1]:

$$H(R^B|R^A) \geq \sum_{\lambda} P(\lambda) H_q(R^B|\lambda)$$

EPR-steering inequalities (CV)

Because of our LHS constraint

$$h(x^B|x^A) \geq \int d\lambda\, \rho(\lambda) h_q(x^B|\lambda)$$

we can use the uncertainty relation [3],

$$h_q(x^B) + h_q(k^B) \geq \log(\pi e),$$

to get the steering inequality [2],

$$h(x^B|x^A) + h(k^B|k^A) \geq \log(\pi e).$$

EPR-steering inequalities (DV)

Because of our LHS constraint

$$H(R^B|R^A) \geq \sum_{\lambda} P(\lambda) H_q(R^B|\lambda)$$

We can use the uncertainty relation [4],

$$H_q(Q^B) + H_q(R^B) \geq \log(\Omega^B),$$

to get the steering inequality [1]

$$H(R^B|R^A) + H(S^B|S^A) \geq \log(\Omega^B)\,.$$

$$\Omega^B \equiv \min_{i,j}\left(\frac{1}{\left|\langle Q_i^B | R_j^B \rangle\right|^2}\right)$$

Entropic EPR-steering inequalities

- Because LHS constraints deal with only one observable at a time…
- **We can get EPR-steering inequalities from any entropic uncertainty relation.**
  - Between any pair of observables, whether continuous, discrete, or both (e.g. angular position/momentum)
  - Between any complete set of mutually unbiased observables [5]
  - Between pairs of POVMs [6]

Hybrid steering inequalities

- The LHS joint probability doesn't have to be of the same observables

$$\mathrm{P}(L^A, \sigma^B) = \sum_\lambda P(\lambda) P(L^A|\lambda) P_q(\sigma^B|\lambda)$$

$$H(\sigma^B|L^A) \geq \sum_\lambda P(\lambda) H_q(\sigma^B|\lambda)$$

- You can have EPR-steering between disparate degrees of freedom

e.g. (orbital) angular momentum to spin

$$H\left(L_x{}^B\middle|\sigma_x{}^A\right) + H\left(L_z{}^B\middle|\sigma_z{}^A\right) \geq \log(N)$$

$$H\left(\sigma_x{}^B\middle|L_x{}^A\right) + H\left(\sigma_z{}^B\middle|L_z{}^A\right) \geq 1$$

Symmetric EPR-steering inequalities

- Definition: steering inequality whose violation rules out LHS models for both parties.
- Examples:

$$I(R^A\!:R^B) + I(S^A\!:S^B) \leq \max_{A,B} \log\left(\frac{N^2}{\{\Omega^A, \Omega^B\}}\right)$$

$$h(x_A \pm x_B) + h(k_A \mp k_B) \geq \log(\pi e)$$

(for two-qubit systems)

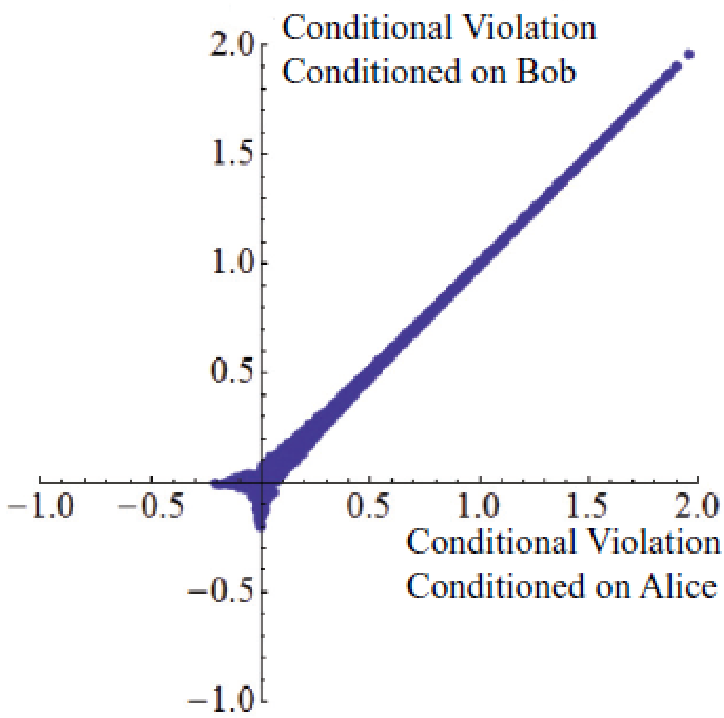
$$I({\sigma_x}^A\!:{\sigma_x}^B) + I\left({\sigma_y}^A\!:{\sigma_y}^B\right) + I({\sigma_z}^A\!:{\sigma_z}^B) \leq 1$$

Steering and QKD

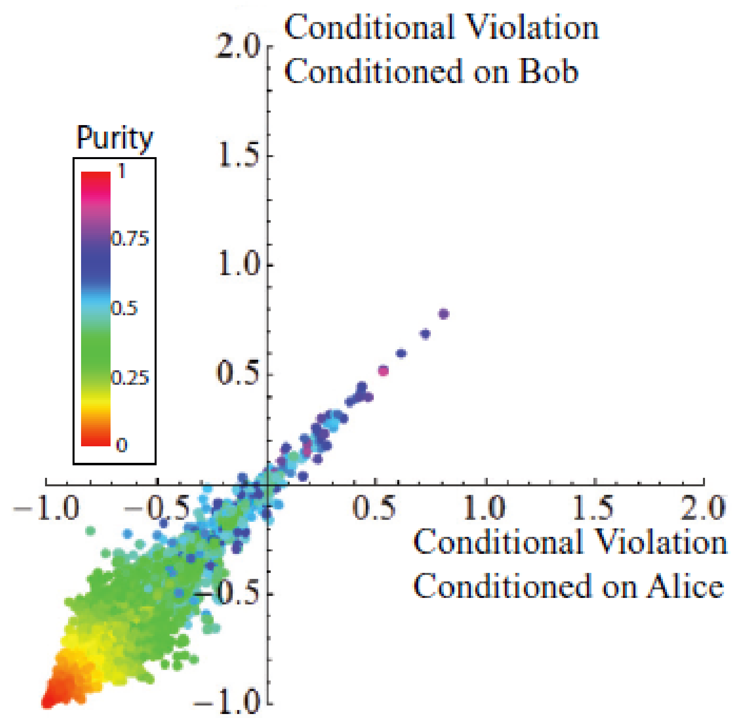
- Symmetrically steerable states guarantee nonzero secret key rate in intercept resend attack.
- Open questions:
  - Do symmetrically steerable states allow some form of device independent QKD?
    - Steerable states allow for one sided device independent QKD [7].
  - Are symmetrically steerable states Bell nonlocal?

Open Question: Are there "one-way" steerable states?

- Definitely maybe!



(a) Scatterplot for optimal pure states



(b) Scatterplot for optimal uniformly sampled two-qubit states

$$H({\sigma_x}^B|{\sigma_x}^A) + H\left({\sigma_y}^B|{\sigma_y}^A\right) + H({\sigma_z}^B|{\sigma_z}^A) \geq 2$$

Conclusion/Related Work

- With any entropic uncertainty relation, we get a viable entanglement witness (practically) for free.

Related work:

- "Continuous variable EPR-steering with discrete measurements": (PRL 110, 130407 (2013)).
- "Quantum Memories and EPR-steering inequalities": (arXiv) (in submission)

Thanks for listening!

DARPA DSO

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Steering with quantum memory?

- Berta *et.al*'s improved uncertainty relation [8]

$$H_q(Q^B) + H_q(R^B) \geq \log(\Omega^B) + S(\hat{\rho}^B)$$

does **not** give us a better steering inequality.

$$\Omega^B \equiv \min_{i,j}\left(\frac{1}{\left|\langle Q_i^B | R_j^B \rangle\right|^2}\right)$$

$$\cancel{\mathrm{H}(R^B|R^A) + H(S^B|S^A) \geq \log(\Omega^B) + S(\hat{\rho}^B)}$$

- Why?